The application of linearization technique for finding drift characteristics of wire detectors

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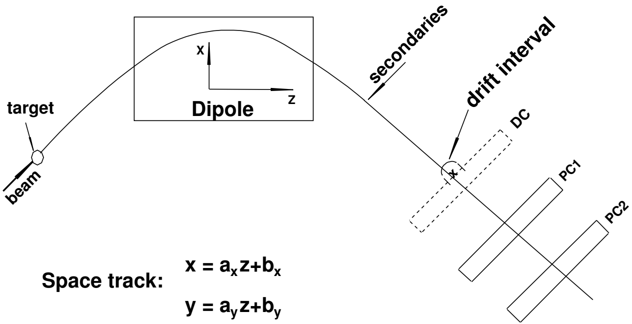
July 7, 2009

Experimental setup



$$\Phi = \sum_{i=1}^{N_t} \left[\sum_{k=1}^{N_{\mathrm{SP}}} \frac{[x_{i,k}^t(\mathbf{a}_i) - x_{i,k}^m]^2}{\sigma_{i,k}^2} + \frac{[\mathrm{xcr}(\mathbf{a}_i) - \mathrm{rt}(\mathbf{p}, t_i)]^2}{\sigma_{\mathrm{DC}}^2} \right] = \min \quad (1)$$

$$\mathbf{a}_i = (a_{i,x}, a_{i,y}, b_{i,x}, b_{i,y})^T, \quad (2)$$

$$\mathbf{p} = (p_1, p_2, \ldots, p_{N_p})^T, \quad (3)$$

where T means matrix transposition, and
$\mathrm{rt}(\mathbf{p}, t_i)$ is range-time relation to be found, $\mathbf{p}$ are parameters;
t_i is the drift time for the i-th track;
N_t is the number of tracks, crossing the drift cell;
$x_{i,k}^t$ are coordinates of crossing of the track with PC_k as a function of $\mathbf{a}_i$;
$x_{i,k}^m$ are measured hit coordinates on the PC's;
$\mathrm{xcr}(\mathbf{a}_i)$ is the distance between crossing of the track with drift chamber and fired wire;
$\sigma_{i,k}$, σ_{DC} are accuracies of PC and DC;
N_p is the size of the vector $\mathbf{p}$; N_{SP} is the number of sensitive planes in PC's

The minimization of the function above can be done by using standard minimization packages like FUMILI[1] or MINUIT[2], but there is a technical restriction: each of these packages while searching for minimum should do inversion of matrices for calculation of step size.
The matrices are symmetric and their dimensions are $(4 \cdot N_t + N_p) \times (4 \cdot N_t + N_p)$. If the number of tracks is ≈ 250, we have a matrix with the number of elements $\approx 1000 \cdot 1000$ and inversion of such a matrix takes a lot of computer time not saying about loosing the accuracy of the calculation. On the other hand to get better accuracy in getting parameters **p** we are interested in using maximum number of tracks (in our case the number of tracks reached $15 \cdot 10^3$). In order to do this we need another approach to finding the minimum. This is exactly a topic of this paper.

Iteration scheme

We have to solve the system of nonlinear equations

$$\frac{d\Phi}{da_{i,j}} = 0, \qquad i = 1, 2, \ldots, N_t, \quad j = 1, 2, 3, 4; \tag{4}$$

$$\frac{d\Phi}{dp_l} = 0, \qquad l = 1, 2, \ldots, N_p, \tag{5}$$

where

$$\frac{d\Phi}{da_{i,j}} = 2\sum_{k=1}^{N_{\mathrm{SP}}} \frac{[x_{i,k}^t(\mathbf{a}_i) - x_{i,k}^m]}{\sigma_{i,k}^2} \cdot \frac{dx_{i,k}^t(\mathbf{a}_i)}{da_{i,j}} + 2 \cdot \frac{[\mathrm{xcr}(\mathbf{a}_i) - \mathrm{rt}(\mathbf{p}, t_i)]}{\sigma_{\mathrm{DC}}^2} \cdot \frac{d(\mathrm{xcr}(\mathbf{a}_i))}{da_{i,j}}$$

and

$$\frac{d\Phi}{dp_l} = -2\sum_{i=1}^{N_t} \frac{[\mathrm{xcr}(\mathbf{a}_i) - \mathrm{rt}(\mathbf{p}, t_i)]}{\sigma_{\mathrm{DC}}^2} \cdot \frac{d(\mathrm{rt}(\mathbf{p}, t_i))}{dp_l}.$$

The system of nonlinear equations (4) and (5) can be solved by Newton method, using iteration procedure with the following linear system for the increments $\Delta a_{i,m}^{[n]}$ and $\Delta p_m^{[n]}$ at each iteration:

$$\sum_{m=1}^{4} \frac{d^2\Phi}{da_{i,j} \cdot da_{i,m}} \cdot \Delta a_{i,m}^{[n]} + \sum_{m=1}^{N_p} \frac{d^2\Phi}{da_{i,j} \cdot dp_m} \cdot \Delta p_m^{[n]} = -\frac{d\Phi}{da_{i,j}}, \tag{6}$$

$$\sum_{m=1}^{4} \frac{d^2\Phi}{dp_l \cdot da_{i,m}} \cdot \Delta a_{i,m}^{[n]} + \sum_{m=1}^{N_p} \frac{d^2\Phi}{dp_l \cdot dp_m} \cdot \Delta p_m^{[n]} = -\frac{d\Phi}{dP_l}, \tag{7}$$

$$a_{i,m}^{[n+1]} = a_{i,m}^{[n]} + \Delta a_{i,m}^{[n]}, \qquad i = 1, 2, \ldots, N_t, \quad m = 1, 2, 3, 4; \tag{8}$$

$$p_m^{[n+1]} = p_m^{[n]} + \Delta p_m^{[n]}, \qquad m = 1, 2, \ldots, N_p, \tag{9}$$

$n = 0,\ 1,\ 2,\ \ldots$

According to linearization scheme [1] for second derivatives of the function Φ, we retain only first order derivatives, namely. So, for elements of Hess matrix we have formulae

$$\frac{d^2\Phi}{da_{i,j}\cdot da_{i,m}} \approx 2\sum_{k=1}^{N_{\mathrm{SP}}} \frac{1}{\sigma_{i,k}^2}\cdot\frac{dx_{i,k}^t(\mathbf{a}_i)}{da_{i,j}}\cdot\frac{dx_{i,k}^t(\mathbf{a}_i)}{da_{i,m}} + \frac{2}{\sigma_{\mathrm{DC}}^2}\cdot\frac{d(\mathrm{xcr}(\mathbf{a}_i)}{da_{i,j}}\cdot\frac{d(\mathrm{xcr}(\mathbf{a}_i))}{da_{i,m}}, \tag{10}$$

$$\frac{d^2\Phi}{da_{i,j}\cdot dp_m} \approx -\frac{2}{\sigma_{\mathrm{DC}}^2}\cdot\frac{d(\mathrm{xcr}(\mathbf{a}_i))}{da_{i,j}}\cdot\frac{d(\mathrm{rt}(\mathbf{p},t_i))}{dp_m}, \tag{11}$$

$$\frac{d^2\Phi}{dp_l\cdot dp_m} \approx \frac{2}{\sigma_{DC}^2}\cdot\sum_{i=1}^{N_t}\frac{d(\mathrm{rt}(\mathbf{p},t_i))}{dp_l}\cdot\frac{d(\mathrm{rt}(\mathbf{p},t_i))}{dp_m}. \tag{12}$$

It is clear that

$$\frac{d^2\Phi}{dp_l\cdot da_{i,m}} = \frac{d^2\Phi}{da_{i,m}\cdot dp_l}. \tag{13}$$

The equations (6), (7) can be rewritten in the form

$$\mathbf{Q}_i \cdot \Delta \mathbf{a}_i^{[n]} + \mathbf{R}_i \cdot \Delta \mathbf{p}^{[n]} = -\mathbf{s}_i, \qquad i = 1, 2, \ldots, N_t; \tag{14}$$

$$\sum_{i=1}^{N_t} \mathbf{R}_i^T \cdot \Delta \mathbf{a}_i^{[n]} + \mathbf{Z} \cdot \Delta \mathbf{p}^{[n]} = -\mathbf{g}, \tag{15}$$

using column vectors of partial gradients

$$\mathbf{s}_i = \left[\frac{d\Phi}{da_{i,1}}, \ldots, \frac{d\Phi}{da_{i,4}} \right]^T, \qquad \mathbf{g} = \left[\frac{d\Phi}{dp_1}, \ldots, \frac{d\Phi}{dp_{N_p}} \right]^T,$$

and matrices

$$\mathbf{Q}_i = \left[\frac{d^2\Phi}{da_{i,j} \cdot da_{i,m}} \right]_{j=\overline{1,4},\, m=\overline{1,4}}, \qquad \mathbf{R}_i = \left[\frac{d^2\Phi}{da_{i,j} \cdot dp_l} \right]_{j=\overline{1,4},\, l=\overline{1,N_p}},$$

$$\mathbf{Z} = \left[\frac{d^2\Phi}{dp_l \cdot dp_m} \right]_{l=\overline{1,N_p},\, m=\overline{1,N_p}}.$$

From equation (14) we can express vector increment $\Delta\mathbf{a}_i^{[n]}$ via $\Delta\mathbf{p}^{[n]}$:

$$\Delta\mathbf{a}_i^{[n]} = -\mathbf{Q}_i^{-1} \cdot [\mathbf{s}_i + \mathbf{R}_i \cdot \Delta\mathbf{p}^{[n]}] = -\mathbf{Q}_i^{-1} \cdot \mathbf{s}_i - \mathbf{Q}_i^{-1} \cdot \mathbf{R}_i \cdot \Delta\mathbf{p}^{[n]}, \quad (16)$$

where $\mathbf{Q}_i^{-1}$ is the inverse of the matrix $\mathbf{Q}_i$. Substituting last expression into (15) we will arrive to the following equation:

$$\mathbf{g} - \sum_{i=1}^{N_t} \mathbf{R}_i^T \cdot \mathbf{Q}_i^{-1} \cdot \mathbf{s}_i = \left[\sum_{i=1}^{N_t} \mathbf{R}_i^T \cdot \mathbf{Q}_i^{-1} \cdot \mathbf{R}_i - \mathbf{Z} \right] \cdot \Delta\mathbf{p}^{[n]}. \quad (17)$$

Then for the increment $\Delta\mathbf{p}^{[n]}$ we obtain a formula

$$\Delta\mathbf{p}^{[n]} = \mathbf{H}^{-1} \cdot \left[\mathbf{g} - \sum_{i=1}^{N_t} \mathbf{R}_i^T \cdot \mathbf{Q}_i^{-1} \cdot \mathbf{s}_i \right], \quad (18)$$

where

$$\mathbf{H} = \sum_{i=1}^{N_t} \mathbf{R}_i^T \cdot \mathbf{Q}_i^{-1} \cdot \mathbf{R}_i - \mathbf{Z}$$

is a matrix of size $N_p \times N_p$.

The iteration scheme looks like follows:

1. We select some initial values for vectors $\mathbf{a}_i^{[0]}$ and $\mathbf{p}^{[0]}$. We put $n = 0$.
2. The increments of $\Delta\mathbf{p}^{[n]}$ are calculated according to formula (18).
3. The increments of $\Delta\mathbf{a}_i^{[n]}$ are calculated according to formula (16).
4. Using increments $\Delta\mathbf{a}_i^{[n]}$ and $\Delta\mathbf{p}^{[n]}$ we get new values for $\mathbf{a}_i$ and $\mathbf{p}$ according (8) and (9). If the precision has not been reached, we increase the counter n, go back to the step 2 and repeat again all the steps 2, 3, and 4.

Convergence of this procedure is controlled by the requirements $\Delta\mathbf{p} \to 0$.

Remark. System (6)–(7) can be rewritten in the block matrix form:

$$\begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{R}_1 \\ \mathbf{0} & \mathbf{Q}_2 & \cdots & \mathbf{0} & \mathbf{R}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{Q}_{N_t} & \mathbf{R}_{N_t} \\ \mathbf{R}_1^T & \mathbf{R}_2^T & \cdots & \mathbf{R}_{N_t}^T & \mathbf{Z} \end{bmatrix} \cdot \begin{bmatrix} \Delta\mathbf{a}_1 \\ \Delta\mathbf{a}_2 \\ \vdots \\ \Delta\mathbf{a}_{N_t} \\ \Delta\mathbf{p} \end{bmatrix} = - \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_{N_t} \\ \mathbf{g} \end{bmatrix}. \qquad (19)$$

Multiplying system (19) from the left side by the matrix

$$\begin{bmatrix} \mathbf{Q}_1^{-1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2^{-1} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{Q}_{N_t}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{I}_{N_p} \end{bmatrix},$$

where $\mathbf{I}_{N_p}$ is the identity matrix of order N_p, we get a system

$$\begin{bmatrix} \mathbf{I}_4 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{Q}_1^{-1}\mathbf{R}_1 \\ \mathbf{0} & \mathbf{I}_4 & \cdots & \mathbf{0} & \mathbf{Q}_2^{-1}\mathbf{R}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_4 & \mathbf{Q}_{N_t}^{-1}\mathbf{R}_{N_t} \\ \mathbf{R}_1^T & \mathbf{R}_2^T & \cdots & \mathbf{R}_{N_t}^T & \mathbf{Z} \end{bmatrix} \cdot \begin{bmatrix} \Delta\mathbf{a}_1 \\ \Delta\mathbf{a}_2 \\ \vdots \\ \Delta\mathbf{a}_{N_t} \\ \Delta\mathbf{p} \end{bmatrix} = - \begin{bmatrix} \mathbf{Q}_1^{-1}\mathbf{s}_1 \\ \mathbf{Q}_2^{-1}\mathbf{s}_2 \\ \vdots \\ \mathbf{Q}_{N_t}^{-1}\mathbf{s}_{N_t} \\ \mathbf{g} \end{bmatrix}. \quad (20)$$

Next multiplying the system (20) from the left side by the matrix

$$\begin{bmatrix} \mathbf{I}_4 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_4 & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_4 & \mathbf{0} \\ -\mathbf{R}_1^T & -\mathbf{R}_2^T & \cdots & -\mathbf{R}_{N_t}^T & \mathbf{I}_{N_p} \end{bmatrix},$$

we arrive to a system

$$\begin{bmatrix} \mathbf{I}_4 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{Q}_1^{-1}\mathbf{R}_1 \\ \mathbf{0} & \mathbf{I}_4 & \cdots & \mathbf{0} & \mathbf{Q}_2^{-1}\mathbf{R}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_4 & \mathbf{Q}_{N_t}^{-1}\mathbf{R}_{N_t} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{Z} - \sum_{i=1}^{N_t} \mathbf{R}_i^T\mathbf{Q}_i^{-1}\mathbf{R}_i \end{bmatrix} \cdot \begin{bmatrix} \Delta\mathbf{a}_1 \\ \Delta\mathbf{a}_2 \\ \vdots \\ \Delta\mathbf{a}_{N_t} \\ \Delta\mathbf{p} \end{bmatrix} = - \begin{bmatrix} \mathbf{Q}_1^{-1}\mathbf{s}_1 \\ \mathbf{Q}_2^{-1}\mathbf{s}_2 \\ \vdots \\ \mathbf{Q}_{N_t}^{-1}\mathbf{s}_{N_t} \\ \mathbf{g} - \sum_{i=1}^{N_t} \mathbf{R}_i^T\mathbf{Q}_i^{-1}\mathbf{s}_i \end{bmatrix},$$

which is equivalent to the system (16)–(17). So, proposed method is equivalent to the block Gauss elimination method of the solution of the system (19).

Acknowledgement

This work was partially supported by Russian RFBR Grants 08-01-00800-a and 07-01-00738-a.

References

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Thank you